Inflationary magnetogenesis from dynamical gauge couplings

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Abstract

The evolution of the gauge coupling is modelled using a scalar degree of freedom whose homogeneous and inhomogeneous modes evolve from an ordinary inflationary phase to the subsequent epochs. Depending upon various parameters (scalar mass, curvature scale at the end of inflation and at the onset of the radiation epoch), the two-point function of the magnetic inhomogeneities grows during the de Sitter stage of expansion and, consequently, large scale magnetic fields are generated. The requirements coming from inflationary magnetogenesis are examined together with the theoretical constraints stemming from the explicit model of the evolution of the gauge coupling. Galactic magnetogenesis is possible for a wide range of parameters. Inter-galactic and inter-cluster magnetogenesis are also discussed.

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I. INTRODUCTION

The gauge coupling has to be (almost) constant by the time when the Universe is approximately old of one second, namely by the time of big-bang nucleosynthesis (BBN). The abundances of light elements are very sensitive to any departure from the standard cosmological model. Hence, fluctuations in the baryon to photon ratio [1], matter–antimatter domains [2], anisotropies in the expansion of the four space-time dimensions [3], can all be successfully constrained by demanding that the abundances of the light elements are correctly reproduced. Following the same logic the variation in the gauge couplings can also be constrained from BBN [4].

The variation of the gauge coupling may also produce effects upon the epoch at which the baryon asymmetry was formed [5,6] namely the epoch of the electroweak phase transition (EWPT) [7] occurring when the Universe was approximately 10^{-11} sec old. The influence of the variation of the gauge couplings on the generation of the baryon asymmetry of the Universe (BAU) is not direct but it can occur in a specific model. Suppose, for instance, that the gauge coupling evolution is modelled using a scalar field. If the field decays prior to the EWPT the BAU generated after the EWPT will be preserved. However, if the field decays between the EWPT and BBN the generated entropy associated with the decay of the field will have to be constrained. Too much entropy could dilute the BAU and, therefore, a constraint on the produced entropy will translate into a constraint on the scalar mass.

In [8] it was argued that the evolution of the gauge couplings may have interesting effects associated with the generation of magnetic fields. In the present investigation the compatibility of the evolution of the gauge coupling with an ordinary (i.e de Sitter or quasi-de Sitter) stage of inflation will be analyzed. In this sense the present discussion is an extension and a completion of the results reported in [8] where the compatibility of the evolution of the gauge coupling with a de Sitter stage of inflation has not been addressed.

The evolution of the Abelian coupling will be mainly investigated. The non-Abelian couplings will be fixed. This choice is motivated by the remark that the effects associated

with large scale gauge fluctuations are related more with the magnetic component of Abelian fields [9]. Furthermore, the necessity of not erasing the produced BAU (by excessive entropy production) leads to exclude a variability of the gauge coupling during the EWPT taking place (roughly) at 100 GeV [7]. To relax these to assumptions is possible but it is beyond the aim of the present discussion.

Suppose that during a de Sitter stage of expansion the coupling constant of an Abelian gauge field evolves in time. Thus the kinetic term of the gauge field can be written (in four space-time dimensions) as

$$S_{\rm em} = -\frac{1}{4}\int d^4 x\sqrt{-G} f(\phi) F_{\alpha\beta}F^{\alpha\beta}, \tag{1.1}$$

where G is the determinant of the space-time metric, ϕ is a scalar field (which can depend upon space and time) and $g(\phi) = f(\phi)^{-1/2}$ is the coupling [1]. This type of vertex is typical of scalar-tensor theories of gravity [10] and of the low energy string effective action [11]. Early suggestions that the Abelian gauge coupling may change over cosmological times were originally made by Dirac [12] (and subsequently discussed in [13] and in [14]) mainly in the framework of ordinary electromagnetism.

The dynamics of the field ϕ will be described by the action of a minimally coupled (massive) scalar. If the field is displaced from the minimum of its potential during inflation, there will be a phase where the field relaxes. Provided the scalar mass is much smaller than the curvature scale during inflation such a phase could be rather long. During the de Sitter stage, the specific form of the expanding background will dictate, through the equations of motion, the rate of suppression of the amplitude of ϕ.

While the field relaxes towards the minimum of its potential, energy is pumped from the homogeneous mode of ϕ to the gauge field fluctuations. The function $f(\phi)$ can be either an increasing function of ϕ (leading to a decreasing coupling) or a decreasing function of ϕ

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[1] The Heaviside electromagnetic system of units will be used throughout the investigation. The effective "electron" charge will then be given, in the present context, by $e(\phi) = e_1 f(\phi)^{-1/2}$.

(leading to an increasing coupling). In both cases, depending upon the parameters of the model, the two-point correlation function of magnetic inhomogeneities increases during the inflationary stage. This implies that large scale magnetic fields can be potentially generated.

Since the pioneering work of Fermi [15] large-scale magnetic fields are a crucial component of the interstellar and perhaps intergalactic medium [17]. Faraday rotation measurements, Zeeman splitting estimates (when available) and synchrotron emission patterns conspire towards the conclusion that distant galaxies are endowed with a magnetic field of roughly the same strength of the one of the Milky Way [17,18].

Observations of magnetic fields at even larger scales (i.e cluster, inter-cluster) are only at the beginning but, recently, promising measurements have been reported. Abell clusters with strong x-ray emission were studied using a twofold technique [19,20]. From the ROSAT [2] full sky survey the electron density along the line of sight has been determined. Faraday rotation (for the same set of 16 Abell clusters) has been determined through observations at the VLA [3]. The amusing result (confirming previous claims based only on one cluster [21]) is that x-ray bright Abell clusters possess a magnetic field of μ Gauss strength. The existence of strong magnetic fields with coherence scale larger than the galactic one can be of crucial importance for the propagation of high-energy cosmic rays.

The large scale galactic magnetic fields are assumed to be the result of the exponential amplification (due to galactic rotation) of some primeval seed fields [16,22]. It was Harrison [16] who suggested that these seeds might have something to do with cosmology in the same way as he suggested that the primordial spectrum of gravitational potential fluctuations (i.e. the Harrison-Zeldovich spectrum) might be produced in some primordial phase of the

[2] The ROetgen SATellite was flying from June 1991 to February 1999. ROSAT provided a map of the x-ray sky in the range 0.1–2.5 keV.

[3] The Very Large Array telescope is a radio-astronomical facility consisting of 27 parabolic antennas spread around 20 km in the New Mexico desert.

evolution of the Universe. Since then, several mechanisms have been invoked in order to explain the origin of these seeds [23,24] and few of them are compatible with inflationary evolution.

The plan of the present paper is then the following. In Section II the basic ideas concerning the model of evolution of the gauge coupling will be introduced. In Section III bounds coming both from the homogeneous and from the inhomogeneous evolution of ϕ will be described. In Section IV the evolution of the magnetic inhomogeneities will be addressed along the various stages of the model with particular attention to the role of the two-point function. The magnetohydrodynamical (MHD) approach will be generalized. In Section V the large scale magnetic fields produced in the scenario will be estimated. Section VI contains some concluding remarks.

II. BASIC EQUATIONS

A. Preliminaries

Thanks to the high degree of isotropy and homogeneity of the observed Universe, the background geometry can be described using a (conformally flat) Friedmann-Robertson-Walker (FRW) line element

$$ds^2 = G_{\mu\nu} dx^\mu dx^\nu = a^2(\eta)[d\eta^2 - d\vec{x}^2], \tag{2.1}$$

where η is the conformal time coordinate and $G_{\mu\nu}$ is the four-dimensional space-time metric. The cosmic time coordinate (often employed in this investigation) is related to η as $a(\eta)\, d\eta = dt$.

From the anisotropies of the Cosmic Microwave Background (CMB) it is consistent to assume that the Universe underwent a period of inflationary expansion of de Sitter or quasi-de Sitter type. Therefore $a(\eta) \sim -\eta_1/\eta$ during a phase stopping, approximately, when the curvature scale was $H_1 \leq 10^{-6} M_{\rm P}$. For $\eta > \eta_1$ (possibly after a transient period) the

Universe gets dominated by radiation [i.e. $a(\eta) \sim \eta$] and then, after decoupling, by dust matter [i.e. $a(\eta) \sim \eta^2$].

The action describing the dynamics of the (Abelian) gauge coupling in a given background geometry can be parametrised as

$$S = \int d^4x \sqrt{-G}\Big[\frac{1}{2}G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) - \frac{1}{4g^2(\phi)}F_{\mu\nu}F^{\mu\nu} + J_\alpha A^\alpha\Big]. \tag{2.2}$$

In Eq. (2.2) the contribution of a classical (Ohmic) current has been included because of the relevance, for the problem at hand, of dissipative effects associated with the finite value of the conductivity, as it will be discussed in Section IV.

From Eq. (2.2) the equations of motion can be derived

$$\frac{1}{\sqrt{-G}}\partial_\mu\Big[\sqrt{-G}G^{\mu\nu}\partial_\nu\phi\Big] + \frac{\partial V}{\partial\phi} = \frac{1}{2g^3(\phi)}\frac{\partial g}{\partial\phi}F_{\alpha\beta}F^{\alpha\beta} + \frac{\partial J_\alpha}{\partial\phi}A^\alpha, \tag{2.3}$$

$$\frac{1}{\sqrt{-G}}\partial_\alpha\Big[\frac{\sqrt{-G}}{g^2(\phi)}F^{\alpha\beta}\Big] = -J^\beta. \tag{2.4}$$

Since the gauge coupling appears in the Ohmic current, the term $\partial_\phi J_\alpha \neq 0$. Inserting Eq. (2.4) into Eq. (2.3) the explicit dependence upon the Ohmic current can be expressed, in terms of the gauge fields, as suggested in [14]. Namely, using Eq. (2.4) we can write that

$$\sqrt{-G}\frac{\partial J^\alpha}{\partial\phi} \equiv 2\partial_\mu\Big[\frac{\sqrt{-G}}{g^3}\frac{\partial g}{\partial\phi}F^{\mu\alpha}A_\alpha\Big] - \frac{\sqrt{-G}}{g^3}\frac{\partial g}{\partial\phi}F_{\alpha\beta}F^{\alpha\beta}. \tag{2.5}$$

An alternative form of Eq. (2.4) becomes then

$$\frac{1}{\sqrt{-G}}\partial_\mu\Big[\sqrt{-G}G^{\mu\nu}\partial_\nu\phi\Big] + \frac{\partial V}{\partial\phi} = -\frac{1}{2g^3(\phi)}\frac{\partial g(\phi)}{\partial\phi}F_{\alpha\beta}F^{\alpha\beta}. \tag{2.6}$$

If the gauge coupling does not change, the evolution of Abelian gauge fields is conformally invariant. Hence, using the the conformal time coordinate, the appropriately rescaled gauge field amplitudes obey a set of equations which is exactly the one they would obey Minkowski space. In order to simplify the explicit form of the equations of motion the electric and magnetic fields will be rescaled in such a way that the obtained system of equations will reproduce the usual (conformally invariant) system in the limit $g \to$ constant. The rescalings in the fields are

$$\vec{B} = a^2\vec{\mathcal{B}}, \quad \vec{E} = a^2\vec{\mathcal{E}},$$
$$\vec{A} = a\vec{\mathcal{A}} \quad \vec{J} = a^3\vec{j}, \quad \sigma = \sigma_c a, \tag{2.7}$$

where $\vec{\mathcal{B}}$, $\vec{\mathcal{E}}$, $\vec{\mathcal{A}}$, $\vec{j}$, σ_c are the flat-space quantities whereas $\vec{B}$, $\vec{E}$, $\vec{A}$, $\vec{J}$, σ are the curved-space ones.

Bearing in mind Eq. (2.7), the explicit form of Eqs. (3.10)–(2.6) becomes:

$$\frac{\partial^2\phi}{\partial\eta^2} + 2\mathcal{H}\frac{\partial\phi}{\partial\eta} + \frac{\partial V}{\partial\phi}a^2 - \nabla^2\phi = -\frac{1}{2g^3a^2}\frac{\partial g}{\partial\phi}\Big[\vec{E}^2 - \vec{B}^2\Big], \tag{2.8}$$
$$\frac{\partial\vec{B}}{\partial\eta} = -\vec{\nabla}\times\vec{E}, \tag{2.9}$$
$$\frac{\partial}{\partial\eta}\Big[\frac{1}{g^2(\phi)}\vec{E}\Big] + \vec{J} = \frac{1}{g^2(\phi)}\Big[\vec{\nabla}\times\vec{B} - \frac{2}{g}\frac{\partial g}{\partial\phi}\vec{\nabla}\phi\times\vec{B}\Big], \tag{2.10}$$
$$\vec{\nabla}\cdot\vec{B} = 0, \tag{2.11}$$
$$\vec{\nabla}\cdot\vec{E} = \frac{2}{g}\frac{\partial g}{\partial\phi}\vec{E}\cdot\vec{\nabla}\phi, \tag{2.12}$$
$$\vec{\nabla}\cdot\vec{J} = 0, \qquad \vec{J} = \sigma(\vec{E} + \vec{v}\times\vec{B}). \tag{2.13}$$

where $\vec{v}$ is the bulk velocity of the plasma. The spatial gradients used in Eqs. (2.8)–(2.13) are defined according to the metric (2.1). In Eq. (2.14) the quantity $\mathcal{H} = \partial\ln a/\partial\eta$ has also been introduced. $\mathcal{H}$ is the Hubble factor in conformal time which is related to the Hubble factor in cosmic time as $\mathcal{H} = aH$ where $H = \partial\ln a/\partial t$.

Once the background geometry is specified we are interested in the situation when the gauge field background is vanishing and the only fluctuations are the ones associate with the vacuum state of the Abelian gauge fields. Hence, Eqs. (2.8)–(2.13) allow to compute the evolution of ϕ and the associated evolution of the two-point function of the gauge field fluctuations.

Suppose that ϕ is originally displaced from the minimum of its potential. As far as the zero mode of ϕ is concerned the system of equations can be further simplified:

$$\frac{\partial^2\phi}{\partial\eta^2} + 2\mathcal{H}\frac{\partial\phi}{\partial\eta} + \frac{\partial V}{\partial\phi}a^2 = -\frac{1}{2g^3a^2}\frac{\partial g}{\partial\phi}\Big[\vec{E}^2 - \vec{B}^2\Big], \tag{2.14}$$
$$\frac{\partial\vec{B}}{\partial\eta} = -\vec{\nabla}\times\vec{E}, \tag{2.15}$$

$$\frac{\partial}{\partial\eta}\left[\frac{1}{g^2(\phi)}\vec{E}\right] + \vec{J} = \frac{1}{g^2(\phi)}\vec{\nabla}\times\vec{B}, \tag{2.16}$$

$$\vec{\nabla}\cdot\vec{B} = 0, \quad \vec{\nabla}\cdot\vec{E} = 0, \tag{2.17}$$

$$\vec{\nabla}\cdot\vec{J} = 0, \qquad \vec{J} = \sigma(\vec{E} + \vec{v}\times\vec{B}). \tag{2.18}$$

By now combining together the modified Maxwell's equations we obtain the evolution of the magnetic fields

$$\vec{B}'' - 2\frac{g'}{g}\vec{B}' - \nabla^2\vec{B} = g^2\vec{\nabla}\times\vec{J}, \tag{2.19}$$

where the prime denoted derivation with respect to the conformal time coordinate (the over-dot will denote, instead, derivation with respect to cosmic time).

Once the evolution of the metric is specified, Eq. (2.14) dictates a specific evolution for ϕ and the evolution of ϕ will determine, in its turn, the evolution of the gauge fields. The interesting initial conditions for the system are the ones where the classical gauge field background vanishes. Thus, when the homogeneous component of ϕ starts its evolution during the de Sitter phase, quantum mechanical fluctuations will be postulated as initial conditions of gauge inhomogeneities.

When the background geometry evolves from the de Sitter phase to the subsequent epoch, massive quanta of ϕ are produced. The amount of the produced inhomogeneous modes of ϕ can be computed and it will be shown that the associated energy density will always be smaller than the one of the homogeneous mode. This analysis will be one of the subjects discussed in Section III.

III. CONSTRAINTS ON THE EVOLUTION OF THE GAUGE COUPLING

Sub-millimiter tests of the Newton's law show that no deviations are observed down to distances as small as 0.1 mm [26]. Therefore, ϕ should be massive. Of course the potential of ϕ may be much more complicated than the one provided by a simple mass term. However, for sake of simplicity, a massive scalar will be analyzed since, already in this case, interesting

effects can be analyzed. In spite of the fact that this choice is apparently simple, various constraints on the scalar mass appear.

A. Evolution of the homogeneous mode

Suppose that the potential term driving the evolution of the gauge coupling is simply

$$V(\phi) \sim \frac{m^2}{2}\phi^2. \tag{3.1}$$

During the inflationary stage of expansion the scale factor evolves as $a(\eta) = (-\eta_1/\eta)$ for $\eta < -\eta_1$. The evolution of ϕ is obtained by solving

$$\phi'' + \frac{2}{\eta}\phi' + \frac{\mu^2}{\eta^2}\phi = 0, \tag{3.2}$$

where $\mu = m/H_1$ and where the relation $\mathcal{H} \sim \eta^{-1} \sim aH$ has been used. If $\mu \ll 1$, for $\eta < -\eta_1$ the solution of Eq. (3.2) can be written as

$$\phi_{\rm i}(\eta) = \phi_1 - \phi_2\Big(-\frac{\eta}{\eta_1}\Big)^3. \tag{3.3}$$

The end of the inflationary stage of expansion may not be directly followed by the radiation dominated phase. In the intermediate phase the scalar mass is still small than the curvature scale but the curvature decreases, in general, faster than during the inflationary phase since the background is neither of de Sitter nor of quasi-de Sitter type. The evolution of the scale factor can be parametrised as $a(\eta) \sim \eta^{\alpha}$ where, in order to fix the ideas, $\alpha \sim 2$ could be assumed [4]. The evolution of ϕ will simply be

$$\phi_{\rm rh}(\eta) = \phi_1 + \phi_2\Big[\frac{2\alpha - 4}{2\alpha - 1} + \frac{3}{2\alpha - 1}\Big(\frac{\eta}{\eta_1}\Big)^{2\alpha-1}\Big], \quad \eta_1 < \eta < \eta_{\rm r}, \tag{3.4}$$

where the continuity between Eq. (3.3) and Eq. (3.4) has been required, so that $\phi_{\rm i}(-\eta_1) = \phi_{\rm rh}(\eta_1)$ and $\phi_{\rm i}'(-\eta_1) = \phi_{\rm rh}'(\eta_1)$.

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[4]The case $\alpha = 2$ corresponds to a matter-dominated intermediate stage.

After $\eta_{\rm r}$ the background enters a radiation dominated phase and the evolution of ϕ can be explicitly solved in cosmic time. The equation for ϕ , in this phase, is given by

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0, \quad H = \frac{\dot{a}}{a}, \tag{3.5}$$

which in terms $\Phi = a^{\frac{3}{2}}\phi$, becomes

$$\ddot{\Phi} + \Big[m^2 - \frac{3}{2}\dot{H} - \frac{9}{4}H^2\Big]\Phi = 0. \tag{3.6}$$

In the radiation-dominated stage of expansion Eq. (3.6) becomes

$$\ddot{\Phi} + \Big[m^2 + \frac{3}{16t^2}\Big]\Phi = 0, \tag{3.7}$$

whose solution can be written in terms of Bessel functions [27]

$$\Phi(mt) = \sqrt{mt}\Big[AY_{\frac{1}{4}}(mt) + BJ_{\frac{1}{4}}(mt)\Big]. \tag{3.8}$$

For $mt \ll 1$, ϕ has a constant mode and a solution as $t^{-1/2}$. Recalling the relation between cosmic and conformal time and imposing the continuity of ϕ and ϕ' (in η_r) with the solution of Eq. (3.4) the following form can be obtained:

$$\phi_{\rm r}(\eta) = \phi_1 + \phi_2\Big[\Big(\frac{2\alpha-4}{2\alpha-1} + \frac{6(1-\alpha)}{2\alpha-1}\Big(\frac{\eta_1}{\eta_{\rm r}}\Big)^{2\alpha-1}\Big) + 3\Big(\frac{\eta_1}{\eta_{\rm r}}\Big)^{2\alpha-1}\frac{\eta_{\rm r}}{\eta}\Big], \tag{3.9}$$

which is valid for $\eta_r < \eta < \eta_{\rm m}$. The time $\eta_{\rm m}$ marks the moment where $H \sim m$. When $mt > 1$, the regime of coherent oscillations takes over and the solution (3.8) implies that the energy density stored in ϕ decreases as a^{-3}, meaning that $\phi_{\rm c}(\eta) \sim \eta^{-3/2}$. Since the coherent oscillations decrease as a^{-3} there will be a typical curvature scale $H_{\rm c}$ and a typical time $\eta_{\rm c}$ at which the coherent oscillations become dominant with respect to the radiation background. This moment is determined by demanding that

$$H_{\rm r}^2\, M_{\rm P}^2\Big(\frac{a_{\rm r}}{a_{\rm c}}\Big)^4 \simeq m^2\phi_1^2\Big(\frac{a_{\rm m}}{a_{\rm c}}\Big)^3, \tag{3.10}$$

which also implies that

$$H_{\rm c} \sim m\varphi^4, \tag{3.11}$$

where $\varphi = \phi_1/M_{\rm P}$. Eq. (3.11) has been obtained without tuning the asymptotic value of ϕ to the minimum of its potential. If such a tuning is made, the amplitude of oscillations at $\eta_{\rm m}$ will be smaller than ϕ_1 and it will be given, according to Eq. (3.9), by $\phi_2(\eta_1/\eta_{\rm r})^{2\alpha-1}$. Thus, the scale H_c will be defined by a different relation namely:

$$m^2\phi_2^2\Big(\frac{H_{\rm r}}{H_1}\Big)^{\frac{2(2\alpha-1)}{\alpha+1}}\Big(\frac{m}{H_{\rm r}}\Big)\Big(\frac{a_{\rm m}}{a_{\rm c}}\Big)^3 \simeq H_{\rm r}^2 M_{\rm P}^2\Big(\frac{a_{\rm r}}{a_{\rm c}}\Big)^4, \tag{3.12}$$

leading, ultimately, to

$$H_{\rm c} = m\Big(\frac{\phi_2}{M_{\rm P}}\Big)^4\Big(\frac{H_{\rm r}}{H_1}\Big)^{\frac{4(2\alpha-1)}{\alpha+1}}\Big(\frac{m}{H_{\rm r}}\Big)^2. \tag{3.13}$$

In the approximation of instantaneous reheating [i.e. $\eta_{\rm r} \sim \eta_1$], $H_{\rm r} \sim H_1$. Therefore, from Eq. (3.13) $H_{\rm c}$ is smaller than the value determined in Eq. (3.11) by a factor $(m/H_1)^2$. In the approximation of matter-dominated reheating (i.e. $\alpha \sim 2$), the result of instantaneous reheating is further suppressed by a factor $(H_{\rm r}/H_1)^4$ as one can easily argue from Eq. (3.13). From Eqs. (3.5)–(3.6), the evolution of ϕ will go as η^{-3} when coherent oscillations start dominating.

In spite of the possible tunings made in the asymptotic values of ϕ, after $\eta_{\rm c}$ there will be a typical time at which the field ϕ will decay. In order not to spoil the light elements produced at the epoch of BBN ϕ has to decay at a scale larger than $H_{\rm ns} \simeq T_{\rm ns}^2/M_{\rm P}$ (where $T_{\rm ns} \simeq$ MeV. Since ϕ is only coupled gravitationally the typical decay scale will be given by comparing the rate with the curvature scale giving that

$$H_{\phi} \sim \Gamma \sim \frac{m^3}{M_{\rm P}^2} > H_{\rm ns}, \tag{3.14}$$

implying that $m > 10^4$ GeV. This requirement also demands that the reheating temperature associated with the decay of ϕ will be larger than the BBN temperature.

In order to illustrate some concrete examples of the various possibilities implied by our considerations, suppose that $m \sim 10^3$ TeV and suppose that the asymptotic value of ϕ is fine-tuned to its minimum. Furthermore, suppose that the reheating is instantaneous. Then, according to the picture which has been presented, inflation stops at a scale $H_1 \sim 10^{13}$ GeV

and ϕ starts oscillating at a curvature scale $H_{\rm m} \sim 10^3$ TeV. The coherent oscillations will then become dominant at a curvature scale $H_{\rm c} \sim 10^{-8}$ GeV (having assumed $\phi_2 \sim M_{\rm P}$). The coherent oscillations of ϕ will last down to $H_\phi \sim 10^{-20}$ GeV. After this moment the Universe will be dominated by the radiation produced in the decay of ϕ. Notice, for comparison, that the BBN curvature scale is $H_{\rm ns} \simeq 10^{-25}$ GeV so that the decay occurs well before BBN (five orders of magnitude in curvature scale).

Another illustrative example is the one where $m \sim 10^6$ TeV. In this case the decay of ϕ occurs prior to the EWPT epoch, namely

$$H_\phi > H_{\rm ew}. \tag{3.15}$$

In fact $H_{\rm ew} = \sqrt{N_{\rm eff}} T_{\rm ew}^2/M_{\rm P} \sim 10^{-17}$ GeV (with $N_{\rm eff} = 106.75$ and $T_{\rm ew} \sim 100$ GeV) whereas, from Eq. (3.14), $H_\phi \sim 10^{-9}$ GeV. In more general terms we can say that in order to have the ϕ decay occurring prior to the EWPT epoch we have to demand that $H_\phi > H_{\rm ew}$ which means that $m > 10^5$ TeV.

In closing this section two general comments are in order. If no fine-tuning is made in the asymptotic amplitude of ϕ, the typical scale of the coherent oscillations will almost coincide with m. However, the possibility $\varphi \ll 1$ is still left if, for some reason, we want $H_{\rm c} \ll m$.

The decay of ϕ and the consequent freezing of the gauge coupling should occur prior to the EWPT epoch and the baryon number should be generated, in the present context, at the electroweak time [5,6]. Suppose, for example, that this is not the case and that the BAU has been created prior to the electroweak scale. Suppose, moreover, that the decay of ϕ occurs after baryogenesis. Then the temperature of the radiation gas before the decay of ϕ will be $T_\phi \sim T_{\rm m}(a_{\rm m}/a_\phi) \sim m(m/M_{\rm P})^5$. Thus, the entropy increase due to the decay of ϕ will be $\Delta S \sim (T_{\rm decay}/T_\phi)^3$ where $T_{\rm decay} \sim \sqrt{H_\phi M_{\rm P}}$. This implies that $\Delta S \sim m/M_{\rm P}$. It has been observed in different contexts that in order to preserve a pre-existing BAU one should have $\Delta S < 10^5$ [29,30]. Thus, this bound would imply $m > 10^{14}$ GeV. This is the reason why the present analysis will assume that the decay of ϕ occurs prior to the electroweak time and that the BAU is generated at the EWPT or shortly after.

B. Evolution of the inhomogeneous modes

When the Universe passes from the inflationary stage to the subsequent radiation dominated expansion, inhomogeneities of the field ϕ are generated. This may invalidate the original assumptions and introduce further complications by adding qualitatively new constraints on the scenario.

It is useful to recall that the inhomogeneities of ϕ can be interpreted, in the framework of second quantization, as quanta of the field ϕ. Hence, the inhomogeneities produced because of the sudden change of the geometry from the de Sitter epoch to the radiation dominated epoch can be counted by estimating the number of quanta produced by the sudden change of the geometry according to the well known techniques of curved space-times [31].

Consider the first order fluctuations of the field ϕ

$$\phi(\vec{x},\eta) = \phi(\eta) + \delta\phi(\vec{x},\eta), \tag{3.16}$$

whose evolution equation is, in Fourier space,

$$\psi'' + 2\mathcal{H}\psi' + [k^2 + m^2 a^2]\psi = 0, \tag{3.17}$$

where $\psi(k,\eta)$ is the Fourier component of $\delta\phi(\vec{x},\eta)$. In order to count the number of quanta produced during the transition of the geometry from the inflationary to the radiation dominated stage of expansion the (canonically normalized) amplitude of fluctuations $\Psi = \psi a$ should be defined so that Eq. (3.17) becomes:

$$\Psi'' + [k^2 + m^2 a^2 - \frac{a''}{a}]\Psi = 0. \tag{3.18}$$

In the de Sitter stage of expansion Eq. (3.18) reduces to

$$\Psi_{\rm i}'' + [k^2 + \frac{\mu^2 - 2}{\eta^2}]\Psi_{\rm i} = 0, \tag{3.19}$$

whereas during the radiation dominated stage of expansion Eq. (3.19) takes the form

$$\Psi_{\rm r}'' + [k^2 + \frac{\mu^2(\eta + 2\eta_1)^2}{\eta_1^4}]\Psi_{\rm r} = 0. \tag{3.20}$$

The solution of Eq. (3.19) (with the correct quantum-mechanical normalization for $\eta \to -\infty$) can be written as

$$\Psi_{\rm i}(\eta) = \frac{1}{\sqrt{2k}}\, p\, \sqrt{-x}\;\; H^{(1)}_{\rho}(-x), \tag{3.21}$$

where $x = k\eta$ and $H^{(1)}_{\nu}$ is the first order Hankel function [27]. In the pure de Sitter case, $\rho = 3/2\sqrt{1-(4/9)\mu^2}$ and since $\mu \ll 1$, $\rho \simeq 3/2$; p is a phase factor which has been chosen in such a way that

$$p = \sqrt{\frac{\pi}{2}}\; e^{i\frac{\pi}{4}(1+2\rho)}. \tag{3.22}$$

With this choice of p we have that $\Psi_{\rm i}(\eta) \sim e^{-ik\eta}/\sqrt{2k}$ for $\eta \to -\infty$.

During the radiation dominated stage of expansion Eq. (3.20) is the equation of parabolic cylinder functions [27]. The solutions turning into positive and negative frequencies for $\eta \to +\infty$ are then

$$\begin{aligned} f_{\rm r}(\eta) &= \frac{1}{(2\gamma)^{1/4}}\, e^{i\frac{\pi}{8}} D_{-iq-\frac{1}{2}}(ie^{-i\frac{\pi}{4}}z), \\ f^{*}_{\rm r}(\eta) &= \frac{1}{(2\gamma)^{1/4}}\, e^{-i\frac{\pi}{8}} D_{iq-\frac{1}{2}}(e^{-i\frac{\pi}{4}}z), \end{aligned} \tag{3.23}$$

where

$$z = \sqrt{2\gamma}(\eta + 2\eta_1), \quad q = \frac{k^2}{2\gamma}, \tag{3.24}$$

and where D_{σ} are the parabolic cylinder functions in the Whittaker's notation. The solution of Eq. (3.20)

$$\Psi_{\rm r}(\eta) = c_{+}(k) g_{\rm r}(\eta) + c_{-}(k) g^{*}_{\rm r}(\eta), \tag{3.25}$$

is given in terms of $c_{+}(k)$ and $c_{-}(k)$ which are the two (complex) Bogoliubov coefficients satisfying $|c_{+}(k)|^2 - |c_{-}|^2 = 1$. In a second quantized approach $|c_{-}(k)|^2$ is the mean number of created quanta, whereas in a semi-classical approach $c_{-}(k)$ can be viewed as the coefficient parametrising the mixing between positive and negative frequency modes. In the case $c_{-}(k) \simeq 0$ no mixing takes place and no amplification is produced. In order to determine $c_{\pm}(k)$, $\Psi_{\rm i}(\eta)$ and $\Psi_{\rm r}(\eta)$ should be continuously matched in $\eta = -\eta_1$, namely

$$\Psi_{\rm i}(-\eta_1) = \Psi_{\rm r}(-\eta_1),$$
$$\Psi_{\rm i}'(-\eta_1) = \Psi_{\rm r}'(-\eta_1), \tag{3.26}$$

By solving this system, an exact expression for the Bogoliubov coefficients is obtained which is, in general a function of two variables : $\mu = m\eta_1$ and $x_1 = k\eta_1$. Since $\mu \ll 1$ the exact result can be expanded, in this limit,

$$c_{+}(k) = \pi e^{i\frac{\pi}{8}}\Big\{\frac{i}{\sqrt{2}\Gamma(\frac{3}{4})}S_2(x_1,\rho)\mu^{-\frac{1}{4}} + \frac{(1+i)}{2\Gamma(\frac{1}{4})}[S_1(x_1,\rho)+S_2(x_1,\rho)]\mu^{\frac{1}{4}}\Big\} + \mathcal{O}(\mu^{\frac{5}{4}}),$$
$$c_{-}(k) = \pi e^{-i\frac{\pi}{8}}\Big\{-\frac{i}{\sqrt{2}\Gamma(\frac{3}{4})}S_2(x_1,\rho)\mu^{-\frac{1}{4}} + \frac{(i-1)}{2\Gamma(\frac{1}{4})}[S_1(x_1,\rho)+S_2(x_1,\rho)]\mu^{\frac{1}{4}}\Big\} + \mathcal{O}(\mu^{\frac{5}{4}}), \tag{3.27}$$

where $S_1(x_1,\rho)$ and $S_2(x_1,\rho)$ contain the explicit dependence upon the Hankel's functions:

$$S_1(x_1,\rho) = e^{i\frac{\pi}{4}(1+2\rho)}H^{(1)}_{\rho}(x_1),$$
$$S_2(x_1,\rho) = \sqrt{x_1}e^{i\frac{\pi}{4}(1+2\rho)}\Big[(\rho+\frac{1}{2})\frac{H^{(1)}_{\rho}(x_1)}{\sqrt{x_1}} - \sqrt{x_1}H^{(1)}_{\rho+1}(x_1)\Big]. \tag{3.28}$$

If $\rho \sim 3/2$ Eq. (3.28) gives

$$c_{+}(k) = e^{i\frac{\pi}{8}}\sqrt{\pi}\Big\{-\frac{x_1^{-\frac{3}{2}}}{2\Gamma(\frac{3}{4})\mu^{\frac{1}{4}}} + \frac{i\,x_1^{-\frac{1}{2}}}{2\Gamma(\frac{3}{4})\mu^{\frac{1}{4}}} + \Big[\frac{1}{2\Gamma(\frac{3}{4})\mu^{\frac{1}{4}}} + \frac{(i-1)\mu^{1/4}}{\sqrt{2}\Gamma(\frac{1}{4})}\Big]\sqrt{x_1}\Big\} + \mathcal{O}(\mu^{\frac{5}{4}}),$$
$$c_{-}(k) = e^{-i\frac{\pi}{8}}\sqrt{\pi}\Big\{\frac{x_1^{-\frac{3}{2}}}{2\Gamma(\frac{3}{4})\mu^{\frac{1}{4}}} - \frac{i\,x_1^{-\frac{1}{2}}}{2\Gamma(\frac{3}{4})\mu^{\frac{1}{4}}} + \Big[-\frac{1}{2\Gamma(\frac{3}{4})\mu^{\frac{1}{4}}} + \frac{(i+1)\mu^{1/4}}{\sqrt{2}\Gamma(\frac{1}{4})}\Big]\sqrt{x_1}\Big\} + \mathcal{O}(\mu^{\frac{5}{4}}). \tag{3.29}$$

In the limit $x_1 = k\eta_1 \ll 1$ the mean number of created quanta can be finally approximated as

$$\overline{n}(k) \simeq |c_{-}(k)|^2 = q|k\eta_1|^{-2\rho}\mu^{-1/2} \tag{3.30}$$

where q is a numerical coefficient of the order of 10^{-2}. The energy density of the created (massive) quanta can be estimated from

$$d\rho_{\psi} = \frac{d^3\omega}{(2\pi)^3}\, m\,\overline{n}(k) \tag{3.31}$$

where $\omega = k/a$ is the physical momentum. In the case of a de Sitter phase ($\rho = 3/2$) the typical energy density of the produced fluctuations is

$$\rho_{\psi}(\eta) \simeq q\, m\, H_1^3 \left(\frac{m}{H_1}\right)^{-1/2} \left(\frac{a_1}{a}\right)^3 \tag{3.32}$$

The produced massive quanta may become dominant. If they become dominant after ϕ already decayed they will not lead to further constraints on the scenario. If they become dominant prior to the decay of ϕ further constraints may be envisaged. The scale at which the massive fluctuations become dominant with respect to the radiation background can be determined by requiring that $\rho_{\psi}(\eta_*) \simeq \rho_{\gamma}(\eta_*)$ implying that

$$q\, m\, H_1^3 \Big(\frac{m}{H_1}\Big)^{-1/2} \Big(\frac{a_1}{a_*}\Big)^3 \simeq H_1^2\, M_{\rm P}^2 \Big(\frac{a_1}{a_*}\Big)^4, \tag{3.33}$$

which translates into

$$H_* \simeq q^2\, m \epsilon^4, \tag{3.34}$$

where $\epsilon = H_1/M_{\rm P}$. In order to make sure that the non-relativistic modes will become dominant after ϕ already decayed $H_* < H_{\phi}$ should be imposed, that is to say $m > 10^2$ TeV for $\epsilon \sim 10^{-6}$.

The maximum tolerable amount of entropy, in order not to wash-out any preexisting BAU is model-dependent but, in general, $\Delta S < 10^5$ seems to be acceptable [28–30]. Defining T_{ϕ} as the radiation gas already present at the scale H_{ϕ}, the entropy increase from T_{ϕ} to $T_{\rm decay} \simeq \sqrt{H_{\phi} M_{\rm P}}$ is of the order of

$$\Delta S = \Big(\frac{T_{\rm decay}}{T_{\phi}}\Big)^3, \tag{3.35}$$

where

$$T_{\phi} = T_* \Big(\frac{a_*}{a_{\phi}}\Big) \simeq m\, \xi^{1/6}\, \epsilon^{-1/2}, \tag{3.36}$$

where $\xi = m/M_{\rm P}$. Demanding that $\Delta S < 10^5$ implies that

$$\xi > 10^{-10} \epsilon^3. \tag{3.37}$$

Taking, as usual, $\epsilon \simeq 10^{-6}$, $m > 10$ GeV.

Hence, if the constraints pertaining to the homogeneous mode are enforced, the bounds coming from the inhomogeneous modes do not invalidate the conclusions of the analysis. According to the logic expressed in Eq. (3.15), an illustrative example is the case where $m > 10^5$ TeV and the BAU is generated after EWPT. In this case the bounds obtained in the present section are satisfied and the analysis of the evolution of the inhomogeneous modes shows that the qualitatively new bounds introduced in the picture are less constraining than the ones obtained in the analysis of the dynamics of the homogeneous mode.

IV. EVOLUTION OF THE GAUGE FIELD FLUCTUATIONS

The evolution of the field ϕ during and after the de Sitter stage implies, according to Eqs. (3.3)–(3.9), that the two-point function of the gauge field fluctuations may very well grow.

For $\eta < -\eta_1$, a rough approximation suggests that the effect of the ohmic current is not present and the gauge field is in the vacuum state with $k/2$ energy in each of its modes. By promoting the classical fields to quantum mechanical operators we have that the physical polarizations of the magnetic field can be written as

$$\hat{b}_i(\vec{x},\eta) = \int \frac{d^3k}{(2\pi)^{3/2}} \sum_{\alpha} e_i^{\alpha}[\hat{a}_{k,\alpha} b(k\eta) e^{i\vec{k}\cdot\vec{x}} + \hat{a}^{\dagger}_{-k,\alpha} b^*(k\eta) e^{-i\vec{k}\cdot\vec{x}}], \tag{4.1}$$

where $b(k\eta) = B(k\eta)/g(\eta)$ obey the equation

$$b'' + \Big[k^2 - 2\Big(\frac{g'}{g}\Big)^2 + \frac{g''}{g}\Big] b = 0. \tag{4.2}$$

Notice that $b(k\eta)$ are the correct normal modes whose limit (for $\eta \to -\infty$) should be normalized to $\sqrt{k/2}e^{-ik\eta}$. The two-point correlation function of the magnetic fluctuations can then be expressed as

$$\mathcal{G}_{ij}(\vec{r},\eta) \equiv \langle \hat{b}_i(\vec{x},\eta)\hat{b}_j(\vec{x}+\vec{r},\eta)\rangle = \int \frac{d^3k}{(2\pi)^3} P_{ij}(k) b(k,\eta) b^*(k,\eta) e^{i\vec{k}\cdot\vec{r}}, \tag{4.3}$$

where

$$P_{ij}(k) = (\delta_{ij} - \frac{k_i k_j}{k^2}). \tag{4.4}$$

The magnetic energy density, derived from the energy-momentum tensor corresponding to the action of Eq. (2.2), is related to the trace of the correlation function reported in Eq. (4.1) over the physical polarizations:

$$\rho_B(r,\eta) = \int \rho_B(k,\eta) \frac{\sin kr}{kr} \frac{dk}{k} \tag{4.5}$$

where

$$\rho_B(k,\eta) = \frac{1}{\pi^2} k^3 |b(k,\eta)|^2. \tag{4.6}$$

A necessary condition in order to assess that gauge field fluctuations grow during the de Sitter phase is that the two-point function increases in the limit $\eta \to -\eta_1$ [32].

Suppose that the gauge coupling decreases with monotonic dependence upon the field ϕ, namely

$$g(\eta) = (\frac{\phi - \phi_1}{M_{\rm P}})^{\frac{\lambda}{2}}, \ \ \lambda > 0. \tag{4.7}$$

This parametrisation is purely phenomenological, however, it allows to take into account, at once, some physically interesting cases like the one suggested by the low-energy string effective action where, in the limit of $\phi/M_{\rm P} < 1$, $g^2(\phi) \sim \phi$.

Using Eq. (3.3) and Eq. (4.7) into Eq. (4.2) the time evolution of the normal modes of the magnetic field can be found analytically since the specific form of Eq. (4.2) falls in the same category of Eq. (3.19). Hence,

$$b(k,\eta) = N\sqrt{k\eta} H^{(2)}_{\nu}(k\eta), \ \ N = \frac{\sqrt{k\pi}}{2} e^{-i\frac{\pi}{4}(1+2\nu)}, \tag{4.8}$$

with $\nu = (3\lambda+1)/2$ and where $H^{(2)}_{\nu}(k\eta)$ the Hankel function of second kind [27]. Notice that the normalization N has been chosen in such a way that for $\eta \to -\infty$ the correct quantum mechanical normalization is reproduced. Consequently, following Eq. (4.3), the two-point function evolves as

$$\lim_{\eta \to -\eta_1} \mathcal{G}_{ij}(r,\eta) \sim \left|\frac{\eta}{\eta_1}\right|^{-3\lambda}. \tag{4.9}$$

Since the two-point function increases magnetic fluctuations are generated.

For sake of completeness the case of increasing gauge coupling will now be examined using the following phenomenological parameterisation

$$g(\eta) = (\frac{\phi - \phi_1}{M_{\rm P}})^{-\frac{\delta}{2}}, \quad \delta > 0. \tag{4.10}$$

Again different scenarios can be imagined. For instance, one coould argue in favour of scenarios where the gauge coupling depends upon ϕ (or upon η) in a highly non-monotonic way. For the illustrative purposes of the present investigation it is however sufficient to focus the attention on the case of monotonic dependence.

Following now the same steps outlined in the case of decreasing gauge coupling the evolution of the two-point function can be obtained

$$\lim_{\eta \to -\eta_1} \mathcal{G}_{ij}(r,\eta) \sim \left|\frac{\eta}{\eta_1}\right|^{2-3\delta}. \tag{4.11}$$

for $\delta > 1/3$ and

$$\lim_{\eta \to -\eta_1} \mathcal{G}_{ij}(r,\eta) \sim \left|\frac{\eta}{\eta_1}\right|^{3\delta}. \tag{4.12}$$

for $\delta < 1/3$. If $\delta < 1/3$ the correlation function decreases and this signals that large scale magnetic fields are not produced.

The back-reaction of the produced fluctuations can be safely neglected in de Sitter space. Looking at Eqs. (2.3) and (2.14) it can happen that if the magnetic fluctuations grow too much the term at the right hand side will become of the same order of the others. This is not the case. Using the conventions of this Section together with the explicit form of the scale factor in the de Sitter phase it can be shown that

$$\frac{1}{g^3 a^2}\frac{\partial g}{\partial \phi}\vec{B}^2 \sim \left|\frac{\eta}{\eta_1}\right|^{-2\nu} |k\eta_1|^{5-2\nu} \tag{4.13}$$

where ν is determined from the specific power dependence of the coupling as a function of ϕ. Since we are interested in large scale modes we have $k\eta_1 \ll 1$. Therefore the back reaction

effects are relevant towards the end of the de Sitter phase (i.e. $\eta \sim -\eta_1$) and for $k \sim \eta_1^{-1}$, namely exactly for the modes not relevant for the present investigation.

After the end of inflation the onset of the conductivity dominated regime may not be instantaneous. In this case after η_1 the presence of a reheating phase should be taken into account. Suppose, for instance, that $g(\eta)$ decreases according to Eq. (4.7). Suppose, moreover, that during reheating the background is dominated by the coherent oscillations of the inflaton. In this case the effective evolution of the geometry will be dominated by matter with $a(\eta) \sim \eta^2$. According to Eq. (3.4) (with $\alpha \sim 2$), $\phi \sim \eta^{-3}$. The value of the magnetic inhomogeneities at $\eta_{\rm r}$ will be given by solving Eq. (4.2)

$$b(k, \eta_{\rm r}) \sim A_1 \left(\frac{\eta_{\rm r}}{\eta_1}\right)^{\frac{3}{2}\lambda} + A_2 \left(\frac{\eta_{\rm r}}{\eta_1}\right)^{1-\frac{3}{2}\lambda}, \tag{4.14}$$

where A_1 and A_2 are two arbitrary constants. Depending upon the value of λ the fastest growing solution is selected in Eq. (4.14). This phase may induce further amplification on the two-point function since its main effect is to delay the conductivity-dominated regime.

A. Generalized MHD equations

For $\eta > \eta_r$ the role of the Ohmic diffusion becomes important and the evolution of the magnetic inhomogeneities will be described by the MHD equations generalized to the case of time varying gauge coupling.

The ordinary (i.e. fixed coupling) MHD treatment is an effective description valid for length scales larger than the Debye radius and for frequencies smaller than the iono-acoustic frequency. This means that MHD is accurate in reproducing the spectrum of plasma excitations obtained from the full kinetic (Vlasov-Landau) approach but only for sufficiently low frequencies and for sufficiently large scales.

Implicit in the ordinary MHD analysis is the assumption that the plasma has to be electrically neutral ($\vec{\nabla} \cdot \vec{E} = 0$) over length scales larger than the Debye radius. Thus, this system of equations cannot be applied for distances shorter than the Debye radius and

for frequencies larger than the plasma frequency [9] where a kinetic description should be employed.

MHD equations can be derived from a microscopic (kinetic) approach and also from a macroscopic approach where the displacement current is neglected [33]. If the displacement current is neglected the electric field can be expressed using the Ohm law and the magnetic diffusivity equation can be derived

$$\frac{\partial \vec{B}}{\partial \eta} = \vec{\nabla} \times (\vec{v} \times \vec{B}) + \frac{1}{\sigma} \nabla^2 \vec{B}. \tag{4.15}$$

The term containing the bulk velocity field is called dynamo term and it receives contribution provided parity is globally broken over the physical size of the plasma. In Eq. (4.15) the contribution containing the conductivity is usually called magnetic diffusivity term.

In the superconducting (or ideal) approximation the resistivity of the plasma goes to zero and the induced (Ohmic) electric field is orthogonal both to the bulk velocity of the plasma and to the magnetic field [i. e. $\vec{E} \simeq -\vec{v} \times \vec{B}$]. In the real (or resistive) approximation the resistivity may be very small but it is always finite and the Ohmic field can be expressed as

$$\vec{E} \simeq \frac{\vec{\nabla} \times \vec{B}}{\sigma} - \vec{v} \times \vec{B}. \tag{4.16}$$

If the gauge coupling changes with time the system of equations obtained by neglecting the displacement current receives new contributions and the relevant equations can be obtained, in the resistive approximation, from Eqs. (2.14)–(2.18):

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial \eta}, \tag{4.17}$$

$$\vec{E} = \frac{\vec{J}}{\sigma} - \vec{v} \times \vec{B}, \tag{4.18}$$

$$\frac{1}{g^2} \vec{\nabla} \times \vec{B} = \vec{J} - 2 \frac{g'}{2 g^3} \vec{E}, \tag{4.19}$$

Using Eqs. (4.17)–(4.19) the generalized magnetic diffusivity equation can be obtained:

$$(1 - \frac{2}{\sigma\, g^2} \frac{g'}{g}) \frac{\partial \vec{B}}{\partial \eta} = \vec{\nabla} \times (\vec{v} \times \vec{B}) + \frac{1}{\sigma g^2} \nabla^2 \vec{B}. \tag{4.20}$$

Notice that Eq. (4.20) reproduces Eq. (4.15) if $g' \to 0$.

Suppose now that the plasma, whose effective Ohmic description has been presented, is relativistic. In the case when the coupling is constant σ is constant and it is given by

$$\sigma \equiv \sigma_c(\eta) a(\eta), \tag{4.21}$$

where $\sigma_c \sim T/g^2$ scales as the inverse of $a(\eta)$ if the evolution of the Universe is, to a good approximation, adiabatic.

If g is not constant, σ is not constant anymore but it decreases if the gauge coupling increases and, vice versa, it increases if the gauge coupling decreases. In spite of this, in the generalized MHD equations, the combination which appears is always σg^2 which is roughly constant for an adiabatically expanding Universe.

In the approximation of instantaneous reheating, the solution of Eq. (4.20) is given by

$$B(k,\eta) = B(k,\eta_1) e^{-\int \frac{k^2}{\sigma g^2 - 2\frac{g'}{g}} d\eta}. \tag{4.22}$$

According to Eqs. (3.13)–(3.14), in order to get the coupling frozen prior to $H_{\rm ew} \sim 10^{-17}$ GeV, $m > 10^5$ TeV shall be required. If the gauge coupling is always decreasing as a function of η, it can be parametrized by Eq. (4.7). Hence Eq. (4.22) can be evaluated by using the explicit evolution of ϕ as obtained from Eqs. (3.8)–(3.9) implying that $\phi_{\rm r} \sim \eta^{-1}$, $\phi_{\rm m} \sim \eta^{-3/2}$ and $\phi_{\rm c} \sim \eta^{-3}$. The result is

$$B(k,\eta_0) = \mathcal{I}(\eta_1, \eta_{\rm m}, \eta_\phi, \eta_{\rm c}) e^{-\frac{k^2}{\sigma g^2}(\eta_1 + \eta_0)} \tag{4.23}$$

where η_0 is the present time and

$$\mathcal{I}(\eta_1, \eta_{\rm m}, \eta_\phi, \eta_{\rm c}) = [(\frac{\lambda + \sigma g^2 \eta_1}{\lambda + \sigma g^2 \eta_{\rm m}})(\frac{3\lambda + 2\sigma g^2 \eta_{\rm m}}{3\lambda + 2\sigma g^2 \eta_{\rm c}})^{\frac{3}{2}}(\frac{3\lambda + \sigma g^2 \eta_{\rm c}}{3\lambda + \sigma g^2 \eta_\phi})^3]^{-\lambda[\frac{k}{\sigma g^2}]^2}. \tag{4.24}$$

Concerning Eqs. (4.23)–(4.24) few comments are in order. From Eq. (4.23) all the modes

$$k^2 > k_\sigma^2 \sim \frac{\sigma g^2}{\eta_0} \tag{4.25}$$

are suppressed by the effect of the conductivity. The present value of $\omega_\sigma(\eta_0)$ [5] can be

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[5] With $\omega(\eta) \sim k/a(\eta)$ the physical momentum will be denoted.

estimated by recalling that $1/\eta_0 \sim H_0 a_0$ where $H_0 \sim 10^{-61}\, M_{\rm P}$. Thus $\omega_\sigma \sim 10^{-3}$ Hz. Present modes of the magnetic fields are dissipated if $\omega > \omega_\sigma$.

As far as the problem of galactic magnetic fields is concerned, the relevant set of scales range around the Mpc corresponding to present modes of the magnetic field $\omega_{\rm G} \sim 10^{-14}$ Hz, i.e. $\omega_{\rm G} \ll \omega_\sigma$.

V. MAGNETIC FIELD GENERATION

Large scale magnetic fields have been postulated over fifty years ago in the context of cosmic rays. Prior to the development of magnetohydrodynamics there was the common belief that cosmic rays are in equilibrium with the stars (like the sun) [15]. Fermi was probably the first one to realize that if our galaxy has a magnetic field, then cosmic rays could be in equilibrium with the large scale magnetic field of the galaxy. Implicit in the Fermi argument there was also the idea that large scale magnetic fields could also be present in other galaxies and, indeed, Fermi and Chandrasekar [34] tried to develop the first theory of gravitational instability in the presence of large scale magnetic fileds. Today large scale magnetic fields are measured with a number of different techniques. Among them Faraday rotation is certainly the most robust.

A. Generalities

When polarized radiation passes through a cold plasma containing a magnetic field the polarization plane of the incoming radiation gets rotated by an amount which is directly proportional to the Larmor frequency (and then to the magnetic field), to the square of the plasma frequency (and then to the electron density). The quantity which is experimentally measured is the shift in the polarization plane per quadratic interval of wave-length, namely:

$$\mathrm{RM} = \frac{\Delta\chi}{\Delta\lambda^2} = 811.9 \int \left(\frac{n_{\rm e}}{\rm cm^{-3}}\right)\left(\frac{B}{\mu\mathrm{Gauss}}\right) d\mathcal{L},\ \frac{\rm rad}{m^2} \qquad (5.1)$$

where $\mathcal{L} = l/\mathrm{kpc}$ and l is the integration variable running over the line of sight. It is clear from the expression of Eq. (5.1) that an independent estimate of the electron density is needed in order to asses the precise value of the magnetic fields we ought to measure. In the interstellar medium an estimate of the electron density along the line of sight is provided by the so called dispersion measurements [17,18]. Pulsars emit regular pulses of electromagnetic radiation with periods ranging from few seconds to few milliseconds. By comparing the arrival times of different radio-signals at different radio-wavelengths, it is found that they are slightly delayed as they pass through the interstellar medium exactly because electromagnetic waves travel faster in the vacuum than in an ionized medium. From the amount of the delay one can infer the dispersion measurement , namely, $\int n_{\mathrm{e}} dl$. Dividing the rotation measurement by the dispersion measurement the mean magnetic field along the line of sight can be estimated.

Recently magnetic fields have been measured in the intra-cluster medium [19]. ROSAT satellite identified a number of x-rays bright Abell clusters (XRBAC). From the surface brightness of the cluster the thermal electron density can be obtained. Various XRBAC (sixteen) have been monitored also with VLA observations. The clusters have been selected in order to show similar morphological features. From VLA observations the RM has been obtained, and from ROSAT the electron density has been obtained. The results show the presence of large scale magnetic fields (of μGauss strength) in the intra-cluster medium. The possible existence of large scale magnetic fields beyond the galaxy is also rather crucial for the deflection of high energy cosmic rays [35,36].

B. Origin of large-scale magnetic fields

The discovery of large scale magnetic fields in the intra-cluster medium implies some interesting problems for the mechanisms of generation of large scale magnetic fields. Let us consider, first of all, magnetic fields in galaxies. Usually the picture for the formation of galactic magnetic fields is related to the possibility of implementing the dynamo mechanism.

The galaxy rotates with a typical period of 10^8 yrs [17]. By comparing the rotation period with the age of the galaxy (for a Universe with $\Omega_\Lambda \sim 0.7$, $h \sim 0.65$ and $\Omega_{\rm m} \sim 0.3$) the number of rotations performed by the galaxy since its origin is approximately 30. During these 30 rotations the dynamo term of Eq. (4.15) dominates against the magnetic diffusivity term since parity is globally broken over the physical size of the galaxy. As a consequence an instability develops. This instability can be used in order to drive the magnetic field from some small initial condition up to its observed value. Most of the work in the context of the dynamo theory focuses on reproducing the correct features of the magnetic field of our galaxy [17,18]. For instance one could ask the dynamo codes to reproduce the specific ratio between the poloidal and toroidal amplitudes of the magnetic field of the Milky Way.

In spite of these aspects, if the number of rotations of the galaxy is approximately 30, the achievable amplification produced by the dynamo instability can be at most of 10^{13}, i.e. e^{30}. Thus, if the present value of the galactic magnetic field is 10^{-6} Gauss, its value right after the gravitational collapse of the protogalaxy might have been as small as 10^{-19} Gauss over a typical scale of 30–100 kpc.

There is a simple way to relate the value of the magnetic fields right after gravitational collapse to the value of the magnetic field right before gravitational collapse. Since the gravitational collapse occurs at high conductivity the magnetic flux and the magnetic helicity are both conserved. Right before the formation of the galaxy a patch of matter of roughly 1 Mpc collapses by gravitational instability. Right before the collapse the mean energy density of the patch, stored in matter, is of the order of the critical density of the Universe. Right after collapse the mean matter density of the protogalaxy is, approximately, six orders of magnitude larger than the critical density.

Since the physical size of the patch decreases from 1 Mpc to 30 kpc the magnetic field increases, because of flux conservation, of a factor $(\rho_{\rm a}/\rho_{\rm b})^{2/3} \sim 10^4$ where $\rho_{\rm a}$ and $\rho_{\rm b}$ are, respectively the energy densities right after and right before gravitational collapse. Henceforth, the correct initial condition in order to turn on the dynamo instability is $B \sim 10^{-23}$ Gauss over a scale of 1 Mpc, right before gravitational collapse.

Since the flux is conserved the ratio between the physical magnetic energy density and the energy density sitting in radiation is almost constant and therefore, in terms of this quantity (which is only scale dependent but not time dependent), the dynamo requirement can be rephrased as

$$r_{\rm B}(L) = \frac{\rho_{\rm B}(L,\eta)}{\rho_{\gamma}(\eta)} \geq 10^{-34}, \quad L \sim 1\,{\rm Mpc}. \tag{5.2}$$

If the dynamo is not invoked but the galactic magnetic field directly generated through some mechanism the correct value to impose at the onset of gravitational collapse is

$$r_{\rm B}(L) \sim 10^{-8}, \quad L \sim 1\,{\rm Mpc}. \tag{5.3}$$

Clearly even the number given in Eq. (5.2) is, physically, not so small. The magnetic energy density stored in a quantum mechanical fluctuations of 1 Mpc is, in terms of $r_{\rm B}$, 10^{-95}.

The possible applications of dynamo mechanism to clusters is still under debate and it seems more problematic [37]. The typical scale of the gravitational collapse of a cluster is larger (roughly by one order of magnitude) than the scale of gravitational collapse of the protogalaxy. Furthermore, the mean mass density within the Abell radius ($\simeq 1.5 h^{-1}$ Mpc) is roughly 10^{3} larger than the critical density [38]. Consequently, clusters rotate less than galaxies since their origin and the value of $r_{\rm B}(L)$ has to be larger than in the case of galaxies. Since the details of the dynamo mechanism applied to clusters are not clear, at present, it will be required that $r_{\rm B}(L) \gg 10^{-34}$ (for instance $r_{\rm B}(L) \simeq 10^{-20}$), in order to see if in the parameter space of the present model magnetic fields larger than the (galactic) dynamo requirement.

C. Estimates of large scale magnetic fields

In the approximation of instantaneous reheating the typical (present) frequency corresponding to the end of inflation can be computed and it turns out to be

$$\omega_1(\eta_0) \sim z_{\rm dec}^{-1}\, T_{\rm dec}\, \epsilon^{1/2}\, \xi^{1/3} \varphi^{-\frac{2}{3}}. \tag{5.4}$$

Since $T_{\rm dec} \sim 0.26$ eV , $z_{\rm dec}^{-1} T_{\rm dec} \sim 100\,{\rm GHz}$. Eq. (5.4) can be obtained by red-shifting the highest mode, i.e. $\omega_1(\eta_1) \sim H_1$ through the different stages of the evolution of the model, namely, according to Eqs. (3.11) and (3.14), from η_1 down to $\eta_{\rm m}$ and from $\eta_{\rm m}$ down to η_{rmc}. Recall that from $\eta_{\rm c}$ to η_ϕ the Universe is, effectively, matter dominated. The other typical frequencies appearing in the time evolution of the gauge coupling can be written, in units of $\omega_1(\eta_0)$, as

$$\begin{aligned}
&\frac{\omega_{\rm m}(\eta_0)}{\omega_1(\eta_0)} = \epsilon^{-1/2}\ \xi^{-1/2},\\
&\frac{\omega_{\rm c}(\eta_0)}{\omega_1(\eta_0)} = \xi^{1/2}\ \epsilon^{-1/2}\ \varphi,\\
&\frac{\omega_{\phi}(\eta_0)}{\omega_1(\eta_0)} = \epsilon^{-1/2}\ \xi^{7/6}\ \varphi^{2/3},
\end{aligned} \tag{5.5}$$

where, as in the case of Eq. (5.4) all the frequencies are evaluated at the present time.

In the case of decreasing gauge coupling [described by Eq. (4.7)] the amount of generated large scale magnetic field can be estimated from Eqs. (4.2)–(4.8) together with Eqs. (4.20)–(4.23). Bearing in mind that the typical frequency scale corresponding to 1 Mpc is 10^{-14} Hz we have, following the notations of Eq. (5.2)

$$r_B(\omega_{\rm G}) = f(\lambda)\ \epsilon^{\frac{3}{2}\lambda}\ \xi^{\lambda-\frac{4}{3}}\ \varphi^{2\lambda-\frac{8}{3}}\ 10^{-25(4-3\lambda)}\ \mathcal{T}(\omega_{\rm G}), \tag{5.6}$$

where

$$f(\lambda) = \frac{2^{3\lambda-1}}{\pi^3}\Gamma^2\biggl(\frac{3}{2}\lambda + \frac{1}{2}\biggr), \tag{5.7}$$

and

$$\mathcal{T}(\omega_{\rm G}) \simeq e^{-\frac{\omega_{\rm G}^2}{\omega_\sigma^2}}[(\frac{\omega_\phi}{\omega_1})(\frac{\omega_\phi}{\omega_{\rm m}})^{\frac{1}{2}}(\frac{\omega_\phi}{\omega_{\rm c}})^{\frac{3}{2}}]^{-\lambda\frac{\omega_{\rm G}^2}{T_0^2}}, \tag{5.8}$$

which means, using Eqs. (5.4)–(5.5),

$$\mathcal{T}(\omega_{\rm G}) = e^{-\frac{\omega_{\rm G}^2}{\omega_\sigma^2}}[\epsilon^{-1/2}\varphi^{-1}\xi^{5/2}]^{-\lambda\frac{\omega_{\rm G}^2}{T_0^2}}. \tag{5.9}$$

Notice that in Eq. (5.8) is the present CMB temperature.

In order to illustrate the regions of the parameter space where magnetogenesis is possible $\varphi \sim 1$ will be assumed. In Fig. 1 and Fig. 2 the case of decreasing gauge coupling is discussed. In Fig. 1, λ is fixed and the exclusion plot is given in terms of ϵ and ξ. In particular, for illustration, $\lambda = 1$ has been chosen. The choice of $\lambda \sim 1$ implies that $g^2(\phi) \sim \phi$. Such a case is favoured from the tree-level string effective action where the effective coupling can be approximated as $g^2(\phi) \sim \phi/M_{\rm P}$ for $\phi < M_{\rm P}$.

The two vertical lines lines mark, respectively, the bounds coming from BBN [i.e. $\xi > 10^{-15}$, from Eq. (3.14)] and from the electroweak epoch [i.e. $\xi > 10^{-11}$, from Eq. (3.15)]. For consistency with the assumptions of Eqs. (3.2)–(3.3) $m/H_1 \ll 1$, implying $\epsilon \gg \xi$. With the lower dashed line the bound $\epsilon > \xi$ is reported. Notice that the requirement of Eq. (3.37) is not numerically relevant. The upper dashed line is obtained by requiring, according to Eq. (3.34), that $H_{\phi} > H_{*}$. Finally, the full (diagonal) line is derived by imposing on Eqs. (5.6)–(5.9) the dynamo requirement of Eq. (5.2).

In Fig. 2 the value of ξ has been fixed to 10^{-11}, as required by the considerations related to the electroweak epoch and the magnetogenesis region is described in terms of ϵ and λ, both varying over their physical range. The two horizontal lines fix the bounds coming from $\epsilon \leq 10^{-6}$ and from $\epsilon > \xi$. With the dashed line the curve $r_{B}(\omega_{\rm G}) \sim 10^{-20}$ is denoted. Hence, values larger than the dynamo requirement are allowed. This observation may be relevant in the context of magnetic fields associated with clusters. In Fig. 2, λ lies in the range $0 < \lambda < 4/3$. This choice guarantees the growth of the correlation function of the magnetic inhomogeneities during the de Sitter stage.

In order not to conflict with large scale bounds coming from the isotropy of the CMB the energy spectra of the produced gauge field fluctuations have to decay at large distance scales, implying that $0 < \lambda \leq 4/3$. To be compatible with CMB anisotropies $r_{\rm B}(\omega_{\rm dec}) \leq 10^{-10}$ should be imposed (where $\omega_{\rm dec} \simeq 10^{-16}$ Hz). If the condition $0 < \lambda < 4/3$ is enforced, the spectra increase with frequency and the possible bounds coming from the anisotropy of the CMB are satisfied.

An analogous estimate can be obtained in the case of increasing gauge coupling discussed

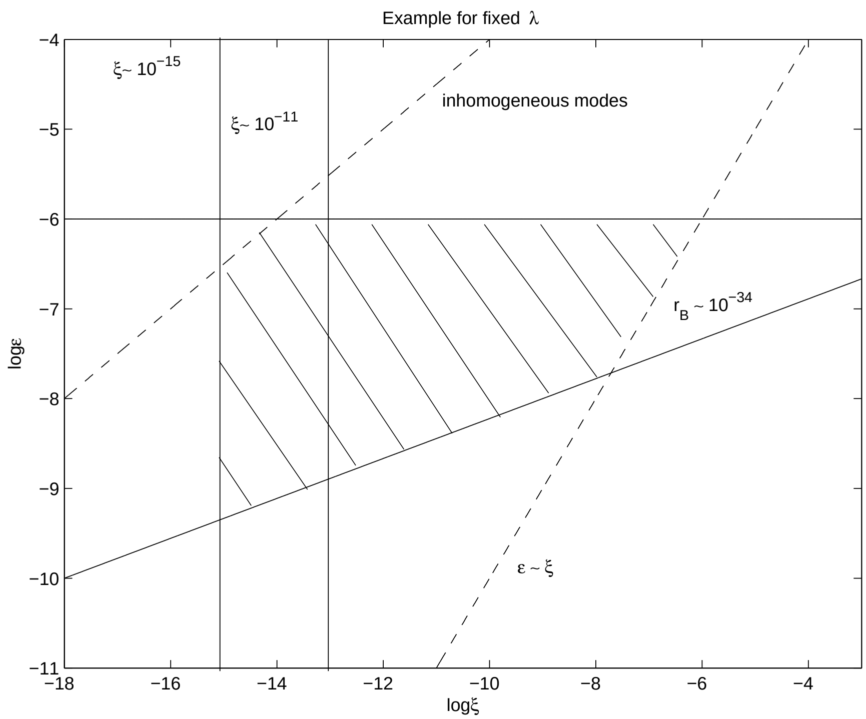

FIG. 1. The shaded area illustrates the region where magnetogenesis is possible in the case where $\lambda = 1$ and $\varphi \sim 1$. The vertical lines correspond to the requirements coming from BBN and from the EWPT epoch.

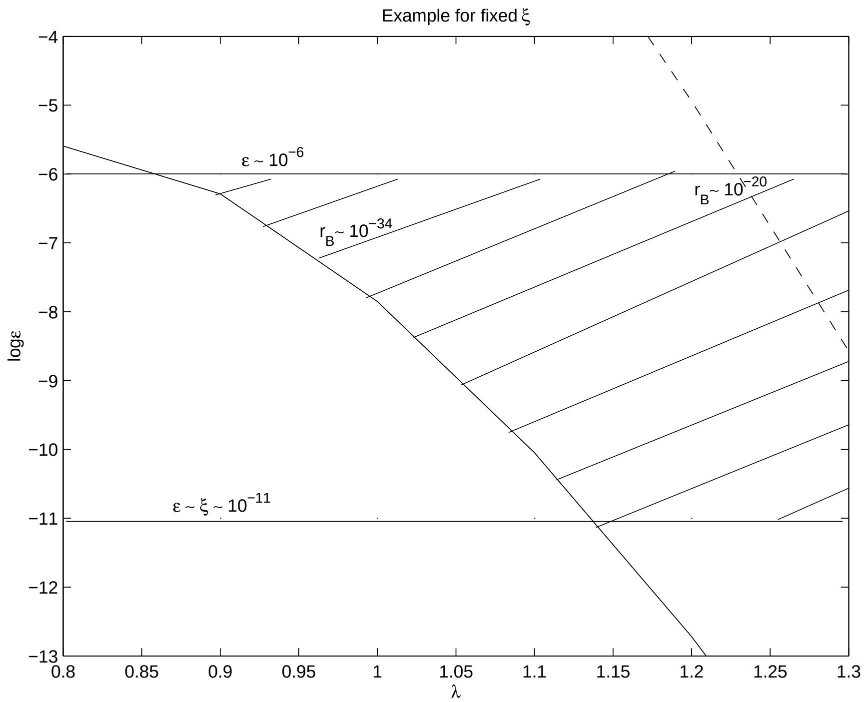

FIG. 2. The region with stripes defines the area where magnetogenesis can occur in the case of fixed mass [i.e. $\xi \sim 10^{-11}$] and for $\varphi \sim 1$.

in Eq. (4.10). In this case

$$r_{\rm B}(\omega_{\rm G}) \sim f(\delta)\epsilon^{\frac{3}{2}\delta-1}\ \xi^{\delta-2}\ \varphi^{4-2\delta}\ 10^{-25(6-3\delta)}\mathcal{T}(\omega_{\rm G}), \tag{5.10}$$

where

$$f(\delta) = \frac{2^{3\delta-3}}{\pi^3}\Gamma^2\Big(\frac{3}{2}\delta - \frac{1}{2}\Big), \tag{5.11}$$

and

$$\mathcal{T}(\omega_{\rm G}) = e^{-\frac{\omega_{\rm G}^2}{\omega_\sigma^2}}\big[\epsilon^{-1/2}\varphi^{-1}\xi^{5/2}\big]^{\delta\frac{\omega_{\rm G}^2}{T_0^2}}. \tag{5.12}$$

In Fig. 3 and Fig. 4 the requirements coming from the dynamo mechanism as well as the other theoretical constraints are illustrated. For both plots $\epsilon < 10^{-6}$ and $\varphi \sim 1$. In Fig. 3 the case $\delta = 2$ is illustrated. The shaded area selects the region of the parameter space where the dynamo requirement of Eq. (5.2) is imposed on Eq. (5.10). The two vertical lines

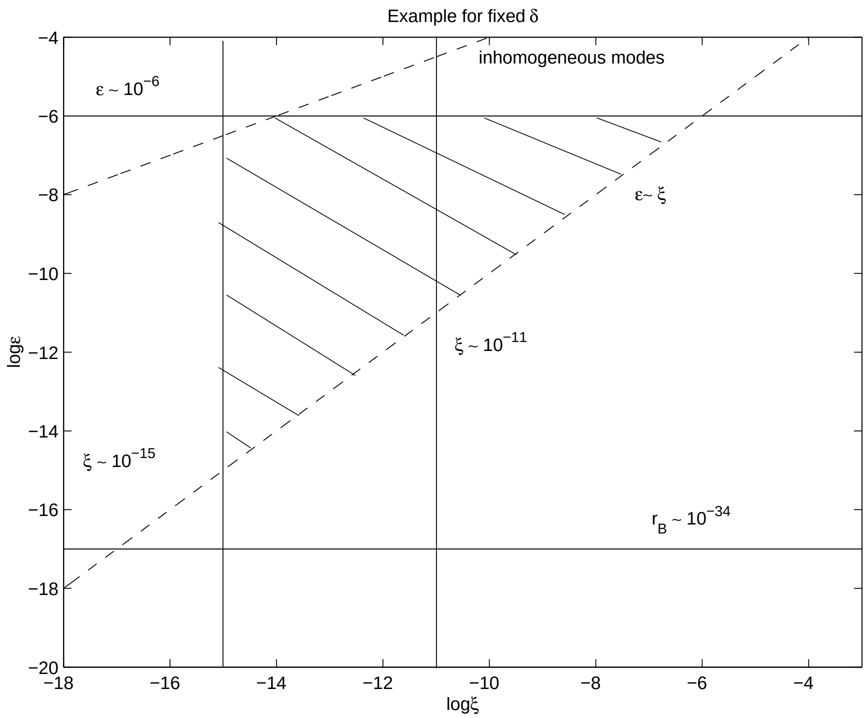

FIG. 3. The magnetogenesis region in the case $\delta = 2$ (i.e. increasing gauge coupling) and $\varphi \sim 1$. The exclusion plot is then given in terms of ϵ and ξ.

illustrate the conditions of Eqs. (3.14)–(3.15). As in Fig. 2 the two dashed lines correspond to the constraints coming from the inhomogeneous modes and from the condition $\epsilon > \xi$.

The requirement that the spectra decrease at large distance scales implies, in this case that $\delta \leq 2$. The condition on the growth of the correlation function obtained in Eqs. (4.11)–(4.12) imply $\delta > 1/3$. Thus the interesting physical range of Eq. (4.10) and (5.10) will be $1/3 < \delta \leq 2$.

In Fig. 4 the parameter space is illustrated for fixed values of ξ, i.e. $\xi \sim 10^{-11}$. As in Fig. 3 the full and dashed curves correspond to the dynamo requirements imposed on Eq. (5.10). The shaded area selects the allowed region in the space of the parameters where magnetogenesis is possible for $10^{-11} < \epsilon < 10^{-6}$. As in the case of Fig. 2 there are regions in the shaded area where values much larger than the magnetogenesis requirement are possible (see the dashed line in Fig. 4).

As it has been pointed out in deriving the theoretical bounds on the scenario the require-

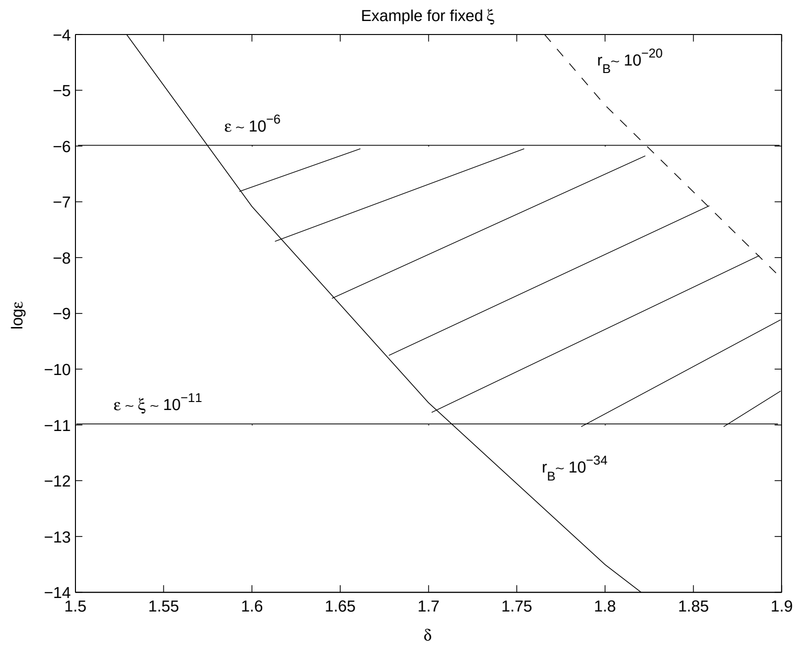

FIG. 4. The magnetogenesis region is illustrated in the case of increasing gauge coupling in the (ϵ, δ) plane. Notice that in the present example $\xi \sim 10^{-11}$ (for compatibility with the EW epoch) and $\varphi \sim 1$.

ment $\xi \geq 10^{-11}$ may be too restrictive since it excludes the variation of the gauge coupling at the electroweak time. To relax this assumption is possible and it would require a precise analysis of the dynamics of the EWPT in the presence of time varying gauge coupling. At the moment this kind of analysis is not available. Summarizing this illustrative discussion, there are regions in the parameter space of the models wher all the theoretical constraints are satisfied and where magnetogenesis is possible. In particular, it is possible that the gauge coupling freezes prior to the electroweak epoch, leading still to magnetogenesis.

VI. CONCLUDING REMARKS

There are no reasons why the gauge couplings should be constant throughout all the history of the Universe. If they are allowed to change prior to the formation of the light elements they can lead to computable differences in the cosmological evolution.

In the present paper the interplay between inflationary magnetogenesis and the evolution

of the Abelian gauge coupling has been addressed. In a phenomenologically reasonable model of inflationary and post-inflationary evolution the relaxation of the gauge coupling leads to a growth in the correlation function of magnetic inhomogeneities. Large scale magnetic fields are then generated. The evolution of the gauge coupling is driven, in the present context, by a massive scalar.

Since the gauge coupling evolves significant changes in the evolution of the plasma can be envisaged. In the present investigation the ordinary MHD equations have been generalized to the case of time evolving gauge coupling but other effects could be envisaged. In particular, the generlization of the full kinetic approach to the case of time evolving "electron" charge would be of related interest.

The value of large scale magnetic fields produced with this mechanism has been estimated. For a broad range of parameters the obtained values of the magnetic fields are much larger than the dynamo requirements.

In the present investigation the main assumption has been that the only gauge coupling free to evolve is the Abelian one. Furthermore, the parameters of the model have been chosen in such a way that the gauge coupling is not dynamical by the onset of the electroweak phase transition. It would be interesting to relax both assumptions since they may lead to potential differences with the standard scenarios.

Therefore, in many respects, the present investigation is not conclusive. At the same time it shows that acceptable models for the evolution of the gauge couplings can be obtained in a standard cosmological framework.

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